The second type are what the authors call index transforms. Again, these transforms have kernels of hypergeometric type, but integration in the inversion formula is performed with respect to a parameter of a hypergeometric function. Two examples are the transform pairs

$$g(y) = \frac{2}{\pi^2} y \sinh \pi y \int_0^\infty \frac{K_{iy}(x)}{x} f(x)\, dx, \qquad f(x) = \int_0^\infty K_{iy}(x)\, g(y)\, dy,$$

(Lebedev), and

$$g(y) = \Gamma(\tfrac{1}{2} - \kappa - iy)\, \Gamma(\tfrac{1}{2} - \kappa + iy) \int_0^\infty W_{\kappa, iy}(x)\, f(x)\, dx,$$

$$f(x) = \frac{1}{(x\pi)^2} \int_0^\infty y \sinh(2\pi y)\, W_{\kappa, iy}(x)\, g(y)\, dy,$$

(Wimp).

The authors of this very welcome book give the first detailed treatment of integral transforms with hypergeometric kernels. They discuss conditions for the validity of inversion formulas, generalized notions of convolution, Parseval type equalities, Erdélyi–Kober fractional operators, convolutional rings, and the application of transforms to the evaluation of integrals and to the solution of integral equations. The book fills a regrettable gap in the mathematical literature. Since Titchmarsh's rather cursory treatment of integral transforms in the book *Theory of Fourier Integrals*, we have lacked any systematic exposition of these exciting and useful ideas.

JET WIMP

G. V. MILOVANOVIĆ, D. S. MITRINOVIĆ, AND TH. M. RASSIAS, *Topics in Polynomials: Extremal Problems, Inequalities, Zeros*, World Scientific, Singapore, 1994, xiii + 821 pp.

> "*Polynomials pervade mathematics and much that is beautiful in mathematics is related to polynomials. Virtually every branch of mathematics from algebraic number theory and algebraic geometry to applied analysis, Fourier analysis, and computer science has its corpus of theory arising from the study of polynomials. Historically questions relating to polynomials, for example, the solution of polynomial equations, gave rise to some of the most important problems of the day. The subject is now much too large to attempt an encyclopedic coverage.*"*

The body of the material the authors selected to explore, focuses on extremal problems and inequalities for polynomials, and properties of the zeros of polynomials. This is a book about classical algebraic and trigonometric polynomials. The discussion does not treat polynomials in an extended sense, and does not cover topics like Chebyshev, Markov, or Descartes systems, Müntz polynomials (or equivalently exponential sums), or rational function spaces. Some classical subjects, such as orthogonal polynomials, are not studied either, partly because their discussion would require separate books, and partly because such books exist.

In the preface the authors write: "*The present book contains some of the most important results on the analysis of polynomials and their derivatives. Besides the fundamental results, which are treated with their proofs, the book also provides an account of the most recent developments concerning extremal properties of polynomials and their derivatives, as well as properties of their zeros. An attempt has been made to present the material in an integrated and self-contained fashion. The book is intended not only for the specialist mathematician, but also for those researchers in the applied and computational sciences who use polynomials as a tool.*"

The topics are tastefully selected and the results are easy to find. Although this book is not really planned as a textbook to teach from, it is excellent for self-study or seminars. This is

* From the preface of the book "*Polynomials and Polynomial Inequalities*" by P. Borwein and T. Erdélyi.

a very useful reference book with many results which have not appeared in a book form yet. It is an important addition to the literature.

Some 1200 references have been cited, including preprints. The references appear at the end of each chapter. At the end of the book a symbol index and a name and subject index are included.

The first chapter reviews some of the classical results on polynomials of one and several variables. The second chapter provides an account of some selected inequalities involving algebraic and trigonometric polynomials. The third chapter studies zeros of polynomials with emphasis on the distribution of the zeros of algebraic polynomials, the Sendov–Iliev conjecture, as well as bounds for the zeros and the number of zeros in certain domains. The Eneström–Kakeya theorem and its various generalizations are also considered. Chapter 4 treats inequalities for trigonometric sums. In addition to classical results, special emphasis is given to the analysis of positivity and monotonicity of certain trigonometric sums and some related orthogonal polynomial sums. The fifth and sixth chapters are devoted to extremal problems for polynomials. In Chapter 5 the classical inequalities of Chebyshev, Markov, Remez, Nikolskii, Bernstein, Korkin, and Zolotarev are studied, which are basic in approximation theory. Results discussed in this chapter also include polynomials with minimal $L_p$ norms, various generalizations of such polynomials, estimates for the coefficients of polynomials, Szegő's extremal problem, incomplete polynomials introduced by G. G. Lorentz, and weighted polynomial inequalities. Chapter 6 presents various Markov- and Bernstein-type inequalities, which are essential in proving inverse theorems of approximation. Markov- and Bernstein-type inequalities for various classes of polynomials with constraints are also presented in detail. The final chapter provides some selected applications of polynomials.

TAMÁS ERDÉLYI

A. L. LEVIN AND D. S. LUBINSKY, *Christoffel Functions and Orthogonal Polynomials for Exponential Weights on* [ −1, 1 ], Memoirs of the American Mathematical Society **535**, Amer. Math. Soc., Providence, RI, 1994, xiii + 146 pp.

For orthogonal polynomials on a bounded interval (without loss of generality we can take [ −1, 1 ]), a very nice theory was developed by G. Szegő, and later generalized by Kolmogorov and Krein, in the first half of this century. Szegő's theory deals with the asymptotic properties of orthogonal polynomials on [ −1, 1 ] with an orthogonality measure $\mu$ such that the Radon–Nikodym derivative $\mu'$ is almost everywhere positive on [ −1, 1 ] and satisfies Szegő's condition

$$\int_{-1}^{1} \log \mu'(x) \frac{dx}{\sqrt{1-x^2}} > -\infty.$$

This condition implies that $\mu'$ is not allowed to be too close to zero on the interval [ −1, 1 ]. Szegő's theory is very powerful, but there exist measures $\mu$ (or weights $w(x)$) on [ −1, 1 ] violating Szegő's condition, such as Pollaczek weights or weights of the form $w(x) = \exp(-(1-x^2)^{-\alpha})$, with $\alpha \geqslant 1/2$. The present monograph gives an extension of the Szegő theory for such weights, in particular for weights $w(x) = e^{-2Q(x)}$, where $Q$ is even and convex in $(-1, 1)$ and grows sufficiently rapidly near $\pm 1$. This means that the monograph under review deals with non-Szegő weights where the violation of Szegő's condition is near the end of the interval [ −1, 1 ]. The essential point is that all interesting features of the weighted polynomials $\sqrt{w(x)}\, p_n(x)$ occur on the Mhaskar-Saff interval $[-a_n, a_n] \subset [-1, 1]$, where $a_n$ is a sequence of numbers tending to one, determined by the weight $Q$. A careful analysis of the orthogonal polynomials on $[-a_n, a_n]$, rather than on [ −1, 1 ], then gives the relevant results presented by the authors. They obtain upper and lower bounds for Christoffel functions,